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The role of *r*-mode damping in the thermal evolution of neutron stars

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ABSTRACT

The thermal evolution of neutron stars (NSs) is investigated by coupling with the evolution of *r*-mode instability that is described by a second order model. The heating effect due to shear viscous damping of the *r*-modes enables us to understand the high temperature of two young pulsars (i.e., PSR B0531+21 and RX J0822-4300) in the framework of the simple *npe* NS model, without superfluidity or exotic particles. Moreover, the light curves predicted by the model within an acceptable parameter regime may probably cover all of the young and middle-aged pulsars in the $\lg T_s^{\infty} - \lg t$ panel, and an artificially strong *p* superfluidity invoked in some early works is not needed here. Additionally, by considering the radiative viscous damping of the *r*-modes, a surprising extra cooling effect is found, which can even exceed the heating effect sometimes although plays an ignorable role in the thermal history.

Key words: stars:neutron - dense matter - stars: evolution

1 INTRODUCTION

The composition of neutron star (NS) interior is still poorly known due to the uncertainties of nuclear physics. It has been hoped that comparing theoretical thermal evolution of NSs with thermal emission data from observations could yield information about their internal properties. Our knowledge of the cooling history of a NS has been improving as we were refining the physical ingredients that play a key role on the thermal evolution of NSs. Page et al. (2004) proposed the minimal cooling model and suggested that four basic physical inputs should be considered in the

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simulation of NS thermal evolution, namely, the equation of state, superfluid properties of relevant components, envelope composition and stellar mass. Aguilera et al. (2008) argued that the magnetic field is another basic physical ingredient should not be ignored. In fact, heating mechanisms should not be ignored, too.

During the evolution of NSs, heating mechanisms may be present and dramatically change the thermal evolution of the star. Several heating mechanisms have been extensively discussed, for example, rotochemical heating (Reisenegger 1995; Fernández & Reisenegger 2005), vortex creep heating (Umeda et al. 1995), Joule heating (Aguilera et al. 2008), heating due to the hardron-quark transition (Kang & Zheng 2007) and heating due to *r*-mode damping (Zheng et al. 2006; Yu et al. 2009).

R-modes in a perfect fluid star with arbitrary rotation arise due to the action of the Coriolis force with positive feedback (Andersson 1998; Friedman & Morsink 1998), succumbing to gravitational radiation-driven Chandrasekhar-Friedman-Schutz instability. However, in a realistic star, the *r*-mode evolution is determined by the competition between damping effect due to viscous dissipation and the destabilizing effect due to gravitational radiation. Based on the conservation of angular momentum, a phenomenological model describing the *r*-mode evolution was proposed by Owen et al. (1998) and improved by Ho & Lai (2000). In this original version of the model, an unbounded growth could lead the modes to an unphysical regime because nonlinear effects are ignored. As an important nonlinear effect, differential rotation induced by *r*-modes was first studied by Rezzolla et al. (2000, 2001) and confirmed by some numerical studies (Stergioulas & Font 2001; Lindblom et al. 2001). Sá (2004) solved the fluid equations within nonlinear theory up to the second order in the mode amplitude and described the differential rotation analytically. By extending the *r*-mode evolution model of Owen et al. (1998) to this nonlinear case, Sá & Tomé (2005, 2006) obtained a saturation amplitude of *r*-modes self-consistently. Yu et al. (2009) studied the long-term spin and thermal evolution of isolated NSs under the influence of the differential rotation. They found that the stars can keep nearly a constant temperature for over a thousand years since the differential rotation can significantly prolong the duration of *r*-modes.

In this paper, we study the thermal evolution of isolated NSs with differential rotation induced by *r*-modes using a realistic equation of state (EOS). The same EOS is used by Kaminker et al. (2001) in the simulation of NS cooling, but their interpretation of the observations requires a strong *p* superfluidity. As pointed by Tsuruta et al. (2002) and Blaschke et al. (2004), the strong *p* superfluidity contradict the microphysical calculations in case of high enough proton concentration to permit nucleon direct Urca process. The inclusion of *r*-mode dissipation would probably resolve

this contradiction. In this work, we also take into account the recently realized radiative viscosity (Sa'd et al. 2009).

After the introduction in this section, Section 2 presents the formulism of r-mode evolution and the spin evolution of NSs up to second order in the mode amplitude, Section 3 displays the equation of thermal evolution and the heating (or cooling) term due to viscous dissipation of r-modes. The physical inputs and our results are given in Section 4, and the conclusions and discussions are presented in the last section.

2 THE *R*-MODE EVOLUTION AND THE SPIN EVOLUTION OF NSS TO SECOND-ORDER

For a rotating barotropic Newtonian star, the r-mode solutions of perturbed fluid equations can be found in spherical coordinates ($r,\ \theta,\ \phi$) at first order in α as (Lindblom et al. 1998),

$$\delta^{(1)}v^r = 0, \tag{1}$$
$$\delta^{(1)}v^\theta = \alpha\Omega C_l l\left(\frac{r}{R}\right)^{l-1}\sin^{l-1}\theta\sin(l\phi+\omega t), \tag{2}$$
$$\delta^{(1)}v^\phi = \alpha\Omega C_l l\left(\frac{r}{R}\right)^{l-1}\sin^{l-2}\theta\cos\theta\cos(l\phi+\omega t), \tag{3}$$

and at second order in α as (Sá 2004)

$$\delta^{(2)}v^r = \delta^{(2)}v^\theta = 0, \tag{4}$$
$$\delta^{(2)}v^\phi = \frac{1}{2}\alpha^2\Omega C_l^2 l^2(l^2-1)\left(\frac{r}{R}\right)^{2l-2}\sin^{2l-4}\theta + \alpha^2\Omega A r^{N-1}\sin^{N-1}\theta, \tag{5}$$

where α represents the amplitude of the oscillation, R and Ω are the radius and angular velocity of the unperturbed star, $\omega = -\Omega(l+2)(l-1)/(l+1)$, $C_l = (2l-1)!!\sqrt{(2l+1)/[2\pi(2l)!l(l+1)]}$, A and N are two constants determined by the initial condition. For simplicity, Sá & Tomé (2005) suggested $N = 2l-1$ and redefined A by introducing a new free parameter K as $A = \frac{1}{2}KC_l^2 l^2(l+1)R^{2-2l}$. For the most unstable $l = 2$ r-mode of primary interest to us, the second-order solution $\delta^{(2)}v^\phi$ shows a differential rotation of the star induced by the r-mode oscillation, i.e., large scale drifts of fluid elements along stellar latitudes.

Using $\delta^{(1)}v^i$ and $\delta^{(2)}v^i$, the corresponding Lagrangian displacements $\xi^{(1)i}$ and $\xi^{(2)i}$ can be derived and then the physical angular momentum of the $l = 2$ r-mode can be calculated up to the second order in α as (Sá 2004; Sá & Tomé 2005)

$$J_r = J^{(1)} + J^{(2)} = \frac{(4K+5)}{2}\alpha^2\tilde{J}MR^2\Omega, \tag{6}$$

where $\tilde{J} = 1.635 \times 10^{-2}$ and

$$J^{(1)} = -\int \rho \partial_\phi \xi^{(1)i} \left(\partial_t \xi_i^{(1)} + v^k \nabla_k \xi_i^{(1)} \right) dV, \tag{7}$$

$$J^{(2)} = \frac{1}{\Omega} \int \rho v^i \left[\partial_t \xi^{(1)k} \nabla_i \xi_k^{(1)} + v^k \nabla_k \xi^{(1)m} \nabla_i \xi_m^{(1)} + \partial_t \xi_i^{(2)} + v^k \left(\nabla_i \xi_k^{(2)} + \nabla_k \xi_i^{(2)} \right) \right] dV. \tag{8}$$

Meanwhile, following Owen et al. (1998) and Sá (2004), we further express the energy of the $l = 2$ r-mode by

$$E_r = J^{(2)}\Omega - \frac{1}{3} J^{(1)}\Omega = \frac{(4K+9)}{2} \alpha^2 \tilde{J} M R^2 \Omega^2. \tag{9}$$

When $K = -2$, $J^{(2)}$ vanishes and the expressions of J_r and E_r return to their canonical forms (Owen et al. 1998), in other words, the differential rotation disappears. Both the physical angular momentum and energy of r-modes are increased by gravitational radiation back reaction and decreased by viscous damping, which yields

$$\frac{dJ_r}{dt} = \frac{2J_r}{\tau_g} - \frac{2J_r}{\tau_v}, \tag{10}$$

$$\frac{dE_r}{dt} = \frac{2E_r}{\tau_g} - \frac{2E_r}{\tau_v}, \tag{11}$$

where τ_g is the growth timescale due to gravitational-wave emission, $\tau_v = (\tau_{sv}^{-1} + \tau_{bv}^{-1} + \tau_{rv}^{-1})^{-1}$ is the damping timescale due to viscous dissipation; τ_{sv},τ_{bv} and τ_{rv} are the timescales of the shear viscous damping, bulk viscous damping and radiative viscous damping (the radiative viscous will interpreted latter), respectively.

From equation (10) and (11), it can be seen that the r-modes are unstable if $(\tau_g^{-1} - \tau_v^{-1})^{-1} > 0$. In this case, a small perturbation would lead to a non-ignorable growth of the modes. The competition between the gravitational destabilizing effect that is dependent on Ω and the T-dependent viscous damping effect determines an instability window in the $T - \Omega$ plane.

For a normal NS with a strong magnetic field ($\sim 10^{10-12}$ G), besides the braking effect due to gravitational radiation, the spindown of the star resulting from magnetic dipole radiation should also be taken into account. So, we ought to write the decrease of the total angular momentum of the star as (Owen et al. 1998; Ho & Lai 2000; Sá & Tomé 2005)

$$\frac{dJ}{dt} = -\frac{3\alpha^2 \tilde{J} M R^2 \Omega}{\tau_g} - \frac{I\Omega}{\tau_m}, \tag{12}$$

where $\tau_m = 1.35 \times 10^9 B_{12}^{-2} (\Omega / \sqrt{\pi G \rho})^{-2}$s is the magnetic braking timescale and $I = \tilde{I} M R^2$ with $\tilde{I} = 0.261$ is the moment of inertial of the star. Due to the r-mode oscillation, the total angular momentum of the star could be separated into two parts, i.e., $J = I\Omega + J_r$. Then, Eqs. (10) and (12)

yield

$$\frac{d\alpha}{dt} = \left[1+\frac{4}{3}(K+2)Q\alpha^2\right]\frac{\alpha}{\tau_g} - \left[1+\frac{1}{3}(4K+5)Q\alpha^2\right]\frac{\alpha}{\tau_v} + \frac{\alpha}{2\tau_m}, \quad (13)$$

$$\frac{d\Omega}{dt} = -\frac{8}{3}(K+2)Q\alpha^2\frac{\Omega}{\tau_g} + \frac{2}{3}(4K+5)Q\alpha^2\frac{\Omega}{\tau_v} - \frac{\Omega}{\tau_m}, \quad (14)$$

where $Q = 3\tilde{J}/2\tilde{I} = 0.094$.

3 THERMAL EVOLUTION OF NSS

The equation of thermal evolution can be written as (Yakovlev et al. 1999; Yakovlev & Pethick 2004)

$$C_V\frac{dT}{dt} = -L_\nu - L_\gamma + H_\nu - \Delta L_\nu, \quad (15)$$

where C_V is the total stellar heat capacity. In NSs composed of simple npe matter, the electrons constitute an almost ideal, strongly degenerate, ultra-relativistic gas; neutrons and protons constitute a non-relativistic strong non-ideal Fermi liquid.

Meanwhile, L_ν is the luminosity of neutrinos generated in numerous reactions in the interiors of neutron stars. The main processes we used are nucleon direct Urca, nucleon modified Urca and nucleon bremsstrahlung. If the proton and electron Fermi momenta are too small compared with neutron Fermi momenta, the nucleon direct Urca process is forbidden because it is impossible to satisfy conservation of momentum (Lattimer et al. 1991). Under typical conditions, one finds that the ratio of the number density of protons to that of nucleons must exceed about 0.11 for the process to be allowed.

L_γ is the surface photon luminosity given by

$$L_\gamma = 4\pi R^2\sigma T_s^4, \quad (16)$$

here σ is the Stefan-Boltzmann constant and T_s is the surface temperature.The relation between T_s and the internal NS temperature T is taken from Potekhin et al. (1997), supposed the outer heat blanketing NS envelope is made of ion and neglecting the effects of surface magnetic fields. We also note that the effective surface temperature detected by a distant observer is $T_s^\infty = T_s\sqrt{1-R_g/R}$, where R_g is the gravitational stellar radius.

$(H_\nu - \Delta L_\nu)$ is the energy per unit time induced by viscous dissipation of r-modes. As we can see from equation (11), one part of the oscillation energy of r-modes is converted into heat energy through shear viscous damping and bulk viscous damping($H_\nu = 2E_r(\tau_{sv}^{-1} + \tau_{bv}^{-1})$), and the other part is converted into neutrino emissivity through radiative viscous damping($\Delta L_\nu = 2E_r\tau_{rv}^{-1}$). As a

result

$$H_\nu - \Delta L_\nu = 2E_r\left(\frac{1}{\tau_{sv}} + \frac{1}{\tau_{bv}} - \frac{1}{\tau_{rv}}\right), \tag{17}$$

using equation (23) below, we get

$$H_\nu - \Delta L_\nu = 2E_r\left(\frac{1}{\tau_{sv}} - \frac{1}{2}\frac{1}{\tau_{bv}}\right). \tag{18}$$

Hence, if $\tau_{sv} < 2\tau_{bv}$, we have $(H_\nu - \Delta L_\nu) > 0$, the star would be heated by viscous dissipation of r-modes ; and if $\tau_{sv} > 2\tau_{bv}$, we have $(H_\nu - \Delta L_\nu) < 0$, viscous dissipation causes an extra cooling of the star.

4 PHYSICAL INPUTS AND RESULTS

4.1 NS EOS

In the simulation of thermal evolution, we employ the simplest possible nuclear composition, namely neutrons (n), protons (p) and electrons (e). We adopt a moderately stiff equation of state (EOS) of this matter proposed by Prakash et al. (1988) (their model I with the compression modulus of saturated nuclear matter $K = 240$MeV). The maximum mass of this model is $M = 1.977M_\odot$, and the direct Urca process is forbidden at $M < M_D = 1.358M_\odot$.

4.2 The r-mode timescales

The calculation of r-mode timescales are very complicated, and until now there are no calculations based on realistic EOS. The following timescales (for $l = 2$ r-modes) we employed in this paper are obtained with a polytropic equation of state as $p = k\rho^2$ for NSs, with k chosen so that the mass and radius of the star are $M = 1.4M_\odot$ and $R = 12.53$ km. In the following equations, the conventions $T_9 \equiv T/10^9$ and $\tilde{\Omega} \equiv \Omega/\sqrt{\pi G\rho}$ are used.

The gravitational radiation timescale is (Owen et al. 1998)

$$\tau_g = 3.26\tilde{\Omega}^{-6}s. \tag{19}$$

The shear viscous damping timescale (due to the neutron-neutron scattering) is (Owen et al. 1998)

$$\tau_{sv} = 2.52 \times 10^8 T_9^2 s. \tag{20}$$

In the case of npe matter where only the modified Urca process is relevant, the bulk viscous damping timescale is (Andersson & Kokkotas 2001)

$$\tau_{bv}(MUrca) = 1.20 \times 10^{11} T_9^{-6} \tilde{\Omega}^{-2} s, \tag{21}$$

and in the case of direct Urca process (Owen et al. 1998)

$$\tau_{bv}(DUrca) = 6.99 \times 10^{8} T_9^{-6} \tilde{\Omega}^{-2} s. \tag{22}$$

Recently, Sa'd et al. (2009) first demonstrated that there exists a new mechanism for damping the energy of stellar oscillations, namely the radiative viscous dissipation. Urca processes contribute to the damping of density perturbations not only by converting energy into heat via bulk viscosity, but also by converting it into an increase of the neutrino emissivity via radiative viscosity. They found the radiative viscosity coefficient is 1.5 times larger than the bulk viscosity coefficient. Thus, the damping time scale of radiative viscosity is

$$\tau_{rv} = \frac{2}{3}\tau_{bv}. \tag{23}$$

Since $\tau_{bv}(MUrca)$ is about three orders larger than $\tau_{bv}(DUrca)$, it is natural that for the two kinds of NSs the direct Urca process is permitted or not, the r-mode instability windows (a window determined by $\tau_g^{-1} - \tau_v^{-1} = 0$ in the $T - \Omega$ plane) are quite different. In contrast, the time evolution behavior of α and Ω for these two different kinds of NSs are similar. We can see this from eqs.(13) and (14) (the following argument is based on the relation $\tau_g << \tau_m$, which is correct unless the magnetic fields of NS is extremely large). For a nascent NS with $T \sim 10^{10}$K and $\Omega \sim \frac{2}{3}\sqrt{\pi G \rho}$, we can easily find $\tau_g << \tau_{sv}$ and $\tau_g << \tau_{bv}$. This means the terms of τ_g in eqs. (13) and (14) are decisive in the early evolution of NS for both cases $\tau_{bv} = \tau_{bv}(DUrca)$ and $\tau_{bv} = \tau_{bv}(MUrca)$. More exactly, fig.3 in Yu et al. (2009) (they adopted $\tau_{bv} = \tau_{bv}(DUrca)$) indicates that not only for a nascent NS but also for NS of phase I, II and III, τ_g dominates. Meanwhile, during the following phase IV and V in their figure, $\tau_{sv} << \tau_{bv}$, τ_g and τ_{sv} control the evolution of NSs. Thus, we conclude that τ_{bv} plays an ignorable role during the evolution of NSs.

4.3 The results

We calculated Equations (13), (14) and (15) numerically, taking the initial temperature $T_0 = 10^{10}$K, the initial r-mode amplitude $\alpha_0 = 10^{-6}$, the initial angular velocity $\Omega_0 = \frac{2}{3}\sqrt{\pi G \rho}$ and the magnetic field $B = 10^{12}$ G.

Fig.1 shows the evolution of α, Ω and T of a $1.4 M_\odot$ NS and $K = 1000$. We can see from Fig.1 (c) that the NS core can keep high temperature for 4.6×10^3(or $10^{3.66}$)years. As illustrated by Yu et al. (2009), during the early part of the r-mode evolution, the rotation energy of the star ($\frac{1}{2} I \Omega^2$) is converted into the oscillation energy, the internal energy, and the energy of gravitational waves.

Nevertheless, during the late part, the energy deposited in the r-modes would be released gradually via heating the star and accelerating the stellar rotation.

Fig.2 (the parameters are the same as Fig.1) displays the evolution curves of $(H_\nu - \Delta L_\nu)$ due to viscous dissipation and $(-L_\nu - L_\gamma)$. The evolution of $(H_\nu - \Delta L_\nu)$ can be divided into three stages: (1) $lgt < -4.61$. In this phase, the relation $\tau_{sv} > 2\tau_{bv}$ is fulfilled, and the viscous dissipation of r-modes results in an extra cooling to the thermal evolution of NS. This extra cooling never plays an important role during the evolution of the NS because it presents in the early part of NS evolution where the star interior is still very hot, and it is too small comparing with the neutrino luminosity L_ν. Moreover, it's easy to understand that the same results can be reached to NS of small mass ($M < 1.358M_\odot$) where the direct Urca process is forbidden. (2) $-4.61 < lgt < 3.65$. In this phase, $\tau_{sv} < 2\tau_{bv}$, the r-mode energy is dissipated mainly by shear viscosity and the star is heated. (3) $lgt > 3.65$. In this phase, $(H_\nu - \Delta L_\nu) = 0$, since the r-modes disappear in this stage (see Fig.1(a)).

Fig.3 shows the surface temperature T_s^∞ evolution of $M = 1.3M_\odot$, $M = 1.365M_\odot$ and $M = 1.4M_\odot$ NS with different K. Note that the direct Urca process is forbidden at $M < M_D = 1.358M_\odot$ in our EOS model. We can see that all curves taking into account r-mode dissipation can explain two young and hot pulsar data (PSR B0531+21 and RX J0822-4300) when taking proper value of K ($K = 100$ for $1.3M_\odot$ and $M = 1.365M_\odot$, $K = 1000$ for $1.4M_\odot$). Maybe the $1.3M_\odot$ ($K = 100$) curve looks too high for the explanation of the two data. While, it should be emphasized we don't expect our model to explain the observation data precisely; because this model still have some uncertainties, such as the realistic non-linear evolution of r-mode instability is not clear and the timescales we used are not based on the specified EOS. Nonetheless, it is certain that the dissipation of the oscillations can provide large enough heat to raise the temperature of young neutron stars, and this isn't a negligible effect.

5. CONCLUSIONS AND DISCUSSIONS

We have studied the thermal evolution of NSs, considering the influence of r-mode instability to second order. For the first time we take into account the radiative viscosity, and find the radiative viscous dissipation of r-modes results in an extra cooling of NSs; while in former studies it is taken for granted that viscous damping would lead to the heating of stars. However, we find that this extra cooling can be well neglected in the thermal evolution history of NSs.

On the other hand, the NS is heated due to shear viscous damping of r-modes, and it can keep a high temperature for several thousand years, even tens of thousands of years. This enables us

to explain two young and hot pulsar data (PSR B0531+21 and RX J0822-4300) with NS model composed of only npe matter, without superfluidity or exotic particles. In contrast, under the same NS EOS, Kaminker et al. (2001) explained these data by the inclusion of strong p superfluidity (the maximum critical temperature $T_{cp} \gtrsim 5 \times 10^9$K). The superfluidity they employed is too strong and it was doubted by many works (Tsuruta et al. 2002; Blaschke et al. 2004); because Takatsuka & Tamagaki (1997) showed, through careful microphysical calculations, that for neutron matter with such high proton concentration as to permit nucleon direct Urca process, the superfluid critical temperature should be extremely low, about several $\times 10^7$K. However, if consider a wider value range of NS mass and K, our light curves may probably cover all of the young and middle-aged thermal emission data, and the artificially strong p superfluidity invoked in Kaminker et al. (2001) is no longer needed . Therefore, the explanation of observation data maybe doesn't contradict the calculation of microphysics. Of course, we don't expect our result to fit the observation data accurately, but future studies using an improved model, or even including other heating effect would further improve our results.

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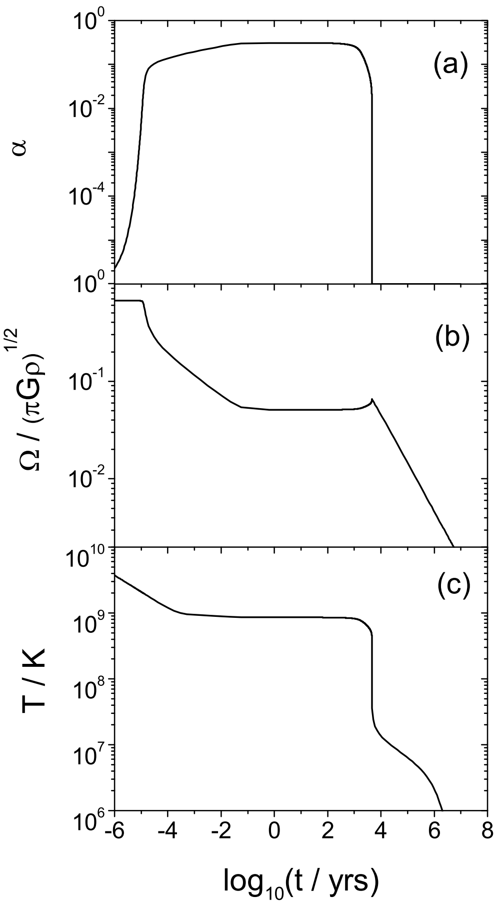

Figure 1. Evolution curves of α, Ω, and T of a $1.4M_{\odot}$ NS with magnetic field $B = 10^{12}$ G and $K = 1000$.

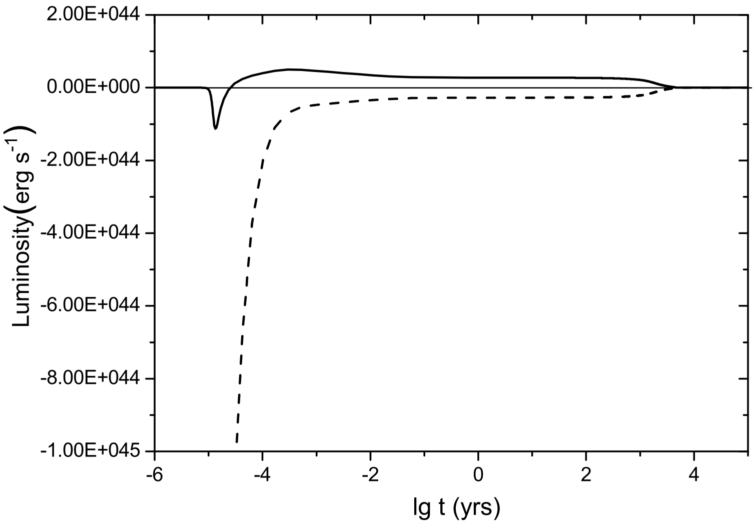

Figure 2. Evolution curves of $(H_\nu - \Delta L_\nu)$ due to viscous dissipation (thick solid line) and $(-L_\nu - L_\gamma)$ (dashed line) with the same parameters as Fig.1. The thin solid one is the zero luminosity line.

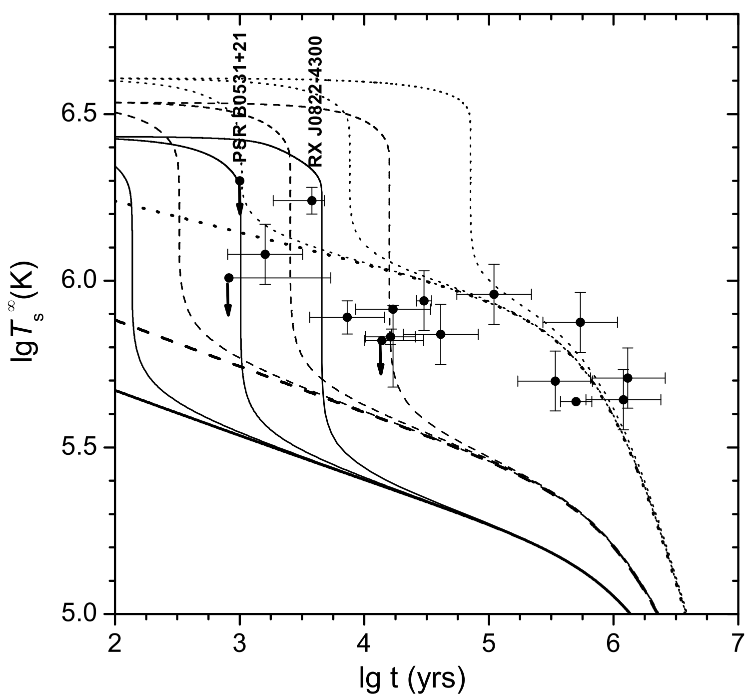

Figure 3. Observational data on the surface temperatures of NSs (Yakovlev et al. 2008) compared with theoretical cooling curves. The dot, dashed and solid curves correspond to $M = 1.3M_\odot$, $M = 1.365M_\odot$ and $M = 1.4M_\odot$, respectively. The thick lines are calculated without the *r*-mode dissipation effect, and the thin lines refer to $K = 10$, 100, 1000 form left to right. For all curves, $B = 10^{12}$G.